2019

Magnetorquer Design I

SFU SATELLITE DESIGN TEAM - ADCS

CALEB GIMPEL

Students at Simon Fraser University are working towards the design of a satellite for the Canadian CubeSat Challenge ending in 2020. The Attitude Determination and Control sub-System are working towards designing and fabricating a magnetorquer which will actuate the satellite in space in a defined slew time to meet the mission requirements. The team design process involves researching magnetorquers and the relevant equations, defining the constraints and selecting materials that will meet the requirements. This is part one of the team design phase and is based strictly on mathematical calculations and models.

Contents

Symbol	Definition	Units
A	Cross-sectional area	m^2
i	Current	A
ρ	Density	Kg / m^3
B_E	Earth's magnetic field	T
μ	Magnetic moment	A·m^2
I	Moment of Inertia	Kg m^2
N	Number of wire turns	-
1U	One CubeSat unit	10cm x 10cm x 10cm
R	Resistance	Ω
τ	Torque	N·m
V	Voltage	V

Abbreviation	Definition
ADCS	Attitude Determination System
CSDC	CubeSat Design Challenge
GPS	Global Positioning System
IGRF	International Geomagnetic Reference Field
LEO	Low Earth Orbit
PCB	Printed Circuit Board

Introduction

Students at Simon Fraser University are designing and building a 3U CubeSat as part of the Canadian Satellite Design Challenge. The payload will require the team to develop its first ever active attitude determination and control system given the requirement to accurately point at a set of coordinates on the Earth in a limited time frame. Frequently, active ADCS systems for CubeSats are purchased in whole or in part from commercial sources; however, since the SFU Satellite Design Team is focused on the advancement of its members education, so they are developing the entire system from the ground up.

The purpose of this paper is to calculate the minimum amount of torque necessary to actuate the satellite in a reasonable amount of time. By defining constraints set by the CSDC competition and the other sub-systems, the final magnetorquer design can be used for preliminary testing and future iterations can be adjusted to consider perturbations.

Attitude Determination and Control System

Attitude Determination and Control System (ADCS) is the subsystem of a satellite design team dedicated to the orientation of the satellite, as well as the position. To determine the orientation and position, a combination of sensors is used to calculate a reliable estimate of its coordinates, which is why sensors are often, but not always, a component when designing the ADCS system.

To calculate the attitude and position, sensors such as sun sensors, global positioning system (GPS), magnetometers and gyroscopes are used together to create an array of processable data. By using a controller algorithm, the position and orientation may be calculated.

Once the position and orientation have been calculated, the satellite may need to reposition to face a desired direction, such as pointing the solar panels towards the sun for recharging or pointing the camera at a set of coordinates on earth. [1] There are a few different types of methods that are used on CubeSats for controlling the attitude, but they all fall under two main brackets: active or passive control system.

An active control system simply means the satellite may be fed commands to activate some actuators which reposition and re-orient the satellite. A passive control system will continuously orient the satellite, but it cannot change its attitude or position based on external feedback. This paper will be focused on the design of a specific type of active actuator: the magnetic torque rod.

Magnetic Torque Rod (Magnetorquers)

Magnetic torque rods, or magnetorquers, are a type of electromagnets composed of a long range of wire looped around a magnetic core. When a current is run through the wire, a magnetic dipole moment is generated which interacts with the external magnetic field of the earth.

The basic law of electromagnetism is defined in equation (1) where $\vec{\mu}$ is the magnetic moment, N is the number of wire turns, i is the current and $\mathbf{A}$ is the cross-section area of the core. The magnetic moment of the magnetorquer is directly proportional to the number of wire turns, cross section of the core and the current.

$$\vec{\mu} = Ni\mathbf{A} \tag{1}$$

By controlling the amount of current running through the torque rod, the magnetic moment may be increased or decreased. If the earth's magnetic field is not parallel to the actuator, the amount of torque generated is defined by the cross product represented in equation (2) where $\vec{\tau}$ is the torque generated and $\vec{B}_E$ is the external magnetic field of the Earth. By changing the magnetic moment vector, the torque induced on the CubeSat may be controlled as well. [2]

$$\vec{\tau} = \vec{\mu} \times \vec{B}_E \tag{2}$$

There are three types of magnetic torquers: solid core, air-core and embedded. For solid and air-core magnetorquers, the long wire is wound around either a solid ferrous material or a vacuum, respectively.

For embedded magnetorquers, the spiral trace is inside the PCB of the solar panels which generates the effect of the coil. Although the embedded option has the least impact on the weight, space and power constraints of the CubeSat, due to the physical limitations it is both difficult to implement and inefficient. Embedded torque rods were not researched further due to the physical limitations.

Air core magnetorquers are a large bundle of wire looped around a cross-sectional area for a given number of turns to generate the torque on the CubeSat. From equation (1) , to generate a large magnetic moment, the number of turns, current and cross-sectional must increase. Without a solid core, the amount of resistance in the torque rod is minimal and hence the power consumption of the air-core torquers is relatively low; however, to achieve higher magnetic moments the cross-sectional area and number of wire turns must be increased to compensate. Since the CubeSats are constrained to a maximum of 1U, the only variable that can be adjusted is the number of wire turns. As the number of wire turns increases, so does the mass.

Solid core magnetorquers are essentially the same as the air-core magnetorquers, but with a magnetic ferrous material that the wire coils around instead of the vacuum. By introducing a ferromagnetic core, the efficiency of the torque rod is increased depending on the type of material used for the core and hence the number of wire turns, and cross-sectional area of the core may be proportionally decreased. [1] [3]

Both the air-core and solid-core magnetorquers have their advantages and disadvantages and it is up to the ADCS team to weigh their trade-offs and decide which type of rod is necessary for the given pay-load. To decide between the two magnetorquer types, the constraints of the payload must be defined.

Moment of Inertia

The moment of inertia is a quantity used to represent a mass's resistance to change in rotation. [4] Just force is proportional to mass times acceleration in linear kinetics, the torque is proportional to the moment of inertia times angular acceleration in rotational kinetics.

$$\vec{\tau} = I\alpha \tag{3}$$

For a simple point mass, the moment of inertia is simply defined as the mass times the square of the distance to the rotation axis; [5] however, depending on the distribution of the mass in the body, the moment of inertia will change as well.

By taking the shape of the body and breaking it down into small pieces, we can sum all these mass points together and multiply them by the square of their distance to the rotation axis. This formula is summarized in (4) where I is the Mass moment of Inertia, r is the distance perpendicular to the axis of rotation and M is the total mass. [5] In equation (4), dm is typically represented by the area of the geometry of the object and the different sections are then added together for the total moment of inertia.

$$I = \int_0^M r^2 \, dm \tag{4}$$

Our CubeSat is a 3U, which is a rectangular shape with the dimensions of 10cm x 10cm x 30cm and the maximum mass is set as 4kg. If we use these dimensions and assume the mass is equally distributed throughout the satellite, then we can assume the moment of inertia will be about its centre of mass at coordinates [0, 0, 0] and the moment of inertia about the x, y and z axis may be represented as follows:

$$I_{Sat} = \begin{vmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{vmatrix} \tag{5}$$

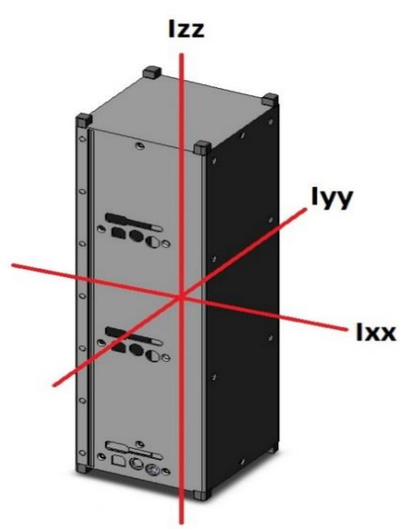

Figure 1 3U CubeSat Moment of Inertia Diagram

From these assumptions, we know the volume V and satellite density ρ to be expressed in (6)

$$\rho = \frac{M}{V}, V{=}xyz \tag{6}$$

The mass elements can then further be expressed as an infinitesimally thin slice based on their density and volume as expressed in (7).

$$dm = \rho dV \tag{7}$$

If we substitute (7) into (4), we get the following general formula, where R is the total length for that axis.

$$I = \int_0^M r^2\, dm = \rho \int_0^R r^2 dV \tag{8}$$

The perpendicular distance to the axis of rotation is given by the Pythagorean theorem [6] where the base width and height are *b, w, h* respectively, and can be substituted for *r* and then separated into two integrals. [7]

$$I_{xx} = \rho \int_{-\frac{y}{2}}^{\frac{y}{2}} \int_{-\frac{z}{2}}^{\frac{z}{2}} (a^2 + b^2) db\, da \tag{9}$$

$$= \rho \int_{-\frac{y}{2}}^{\frac{y}{2}} \left(za^2 + \frac{1}{12} z^3 \right) da$$

$$= \frac{\rho}{12}(y^3 z + yz^3)$$

$$= \frac{M}{12}(y^2 + z^2)$$

From formula (9) we may generalize the mass moment of inertia for our satellite below, where $x = 0.1\text{m}, y = 0.1\text{m}, z = 0.3\text{m}$ and $M = 4\text{kg}$.

$$\mathrm{I_{Sat}} = \begin{vmatrix} \mathrm{I_{xx}} & 0 & 0 \\ 0 & \mathrm{I_{yy}} & 0 \\ 0 & 0 & \mathrm{I_{zz}} \end{vmatrix} = \frac{M}{12} \begin{vmatrix} y^2 + z^2 & 0 & 0 \\ 0 & x^2 + z^2 & 0 \\ 0 & 0 & x^2 + y^2 \end{vmatrix}$$

Axis of Rotation	**Value (kg m^2)**
I_{xx}	0.033333
I_{yy}	0.033333
I_{zz}	0.006667

Table 1 The moment of inertia for a 3U CubeSat

Magnetorquer Design

The design process we used involved setting payload constraints, running our basic calculations, increasing the complexity and revising our assumptions. By iterating through the calculations, it allowed us to continually optimize and redesign for the final selection.

Constraints

Our initial maximum constraints are defined by the CubeSat Design Specification [8] and the rules set by the CSDC [9] and are summarized in the table below:

Parameter	**Value**
Pointing Accuracy	<1.5°
Mass	4kg
Dimensions	10cm x 10cm x 30cm
Required Roll Angle	<25°
Operate LEO	400km-800km
Downlink Time	<90s
Pass time	8min

To achieve a pointing accuracy <1.5°, magnetorquers will not be enough and reaction wheels will be necessary. From these constraints, as well as for design simplicity, the magnetorquer design will be the same for all three axes. Additionally, "The attitude Control subsystem shall be designed so that it is tolerant to at least one attitude actuator failure and will maintain at least a degraded state of performance." [9] Although we will be designing with the intention of introducing reaction wheels, our magnetorquer design will be made such that it could still operate alone.

The maximum satellite mass is defined as 4kg. For our Satellite Design Team, there are 5 sub-teams, and the overall mass divided equally amongst the team is 800gr. For our magnetorquer design, we decided we would design for a constraint of no more than 20% of our maximum sub-team mass budget.

$$800g * 20\% = \frac{160g}{3\ magnetorquers} = \frac{53.\overline{333}g}{magnetorquer}$$

In a single plane, the maximum length a single magnetorquer could be is 10cm as that is the maximum length of the X or Y axis. In practice, the rod must be shorter than 10cm to consider mounting brackets fastened to the ends, but for our calculations we assumed a maximum of 10cm.

The required roll angle is the maximum rotation necessary to point the camera at the target. As this constraint is the minimum roll angle required, our design considered the maximum roll necessary to achieve the same goal. For our design we used a maximum roll angle of 180°.

The CubeSat will be expected to operate in Low Earth Orbit, which is approximately 400km-800km above the earth's surface. Since Low Earth Orbit may still have some atmospheric drag, for our calculations, we must consider the point at which the atmospheric drag is highest: the perigee. [1] For our calculations, we assume the satellite will be operating at the minimum 400km above the earth.

During a Q&A session with the CSDC Management Society for the mission requirements, a single "pass" was undefined. After receiving clarification, a single pass was defined as receiving the Mission Control Centre uplink, rolling into position, taking the image and downlinking the image to the Amateur Radio Operator. Since the downlink time is further constrained to be <1.5min, the rotation has a maximum of 6.5min to complete. For our calculations, we decided to design for the rotation to complete in a maximum of 4 minutes, which allows for delays and error.

Lastly, our design has the assumption the power bus will be 3.3V and the maximum power consumption is 0.2W. These values are provided by the power sub-system.

The individual magnetorquer constraints discussed are summarized in the table below:

Parameter	**Value**
Mass	<53.33g
Length	<10cm
Width	<10cm
Maximum roll	180°
Operate at perigee	400km
Slew time	4min
Voltage	3.3V
Power	0.2W

Table 2 Defined constraints

Moment Calculation

A magnetorquer is defined by the core material, wire material and the number of turns the wire is wrapped around the core, as defined in equation (1). If we can calculate the minimum magnetic moment required to rotate the satellite defined by our constraints, we can extrapolate the necessary parameters to define the magnetorquer. To solve for the minimum magnetic moment, we can use the equation defined in (2).

To start, we must find the magnetic field generated by the earth, or geomagnetic field; however, the earth's magnetic field fluctuates depending on the altitude, time of year and where the satellite is located above the earth. [10] Since the amount of torque generated is proportional to the strength of the magnetic field it is acting upon, the point at which the magnetic field is at its minimum strength will represent the least amount of torque the magnetorquers can generate.

The earth's magnetic field may be simplified by comparing it to a dipole magnet, although it is much more complex and is represented by a spherical harmonic expansion calculated by the International Geomagnetic Reference Field (IGRF). [10] Just as a simple bar magnet generates its magnetic field, the earth's magnetic field leaves the north pole and re-enters at the south pole; however, the magnetic poles are not located at the geographic poles.

The geomagnetic field is weakest when it is furthest from the magnetic poles, called the magnetic equator, specifically in the South Atlantic Anomaly region [11]; if the designed magnetorquers can actuate the satellite at the magnetic equator, it can actuate the satellite at any given point in low earth orbit. For our calculations, the IGRF model provided by the Canadian Government was used to generate a vector representing the geomagnetic field at the magnetic equator. [12]

Earth's Magnetic Field	**Magnitude [T]**
B_{Ex}	27538e-9
B_{Ey}	-2280e-9
B_{Ez}	-16223e-9

Next is the torque vector, which we can find from equation (3) from the moment of inertia and the angular acceleration. Since we have already worked out the moment of inertia, which was summarized in Table 1 The moment of inertia for a 3U CubeSat, we simply need the angular acceleration to calculate the minimum torque which we can get from the basic angular rotation formula.

$$\vec{\theta} = \overrightarrow{\omega_0}t + \frac{1}{2}\vec{\alpha}t^2$$

$$\vec{\alpha} = \frac{2}{t^2}(\vec{\theta} - \overrightarrow{\omega_0}t) \qquad (10)$$

If we assume the satellite has no initial velocity at this point, then then equation for angular acceleration may be further simplified to equation (11). Given this is a simplified model for designing the magnetorquer, this calculation may not be used for detumbling and we must revisit to introduce perturbations for the controller algorithm.

$$\overrightarrow{\alpha_{y,180}} = \frac{2}{t^2}\begin{vmatrix} cos\,\theta & 0 & sin\,\theta \\ 0 & 1 & 0 \\ -sin\,\theta & 0 & cos\,\theta \end{vmatrix} = \frac{2}{t^2}\begin{vmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{vmatrix} \qquad (11)$$

Given our constraints defined for slew time and angle of rotation in Table 2 Defined constraints we can simply plug in the values and find the minimum desired angular rotation to be the following:

Angular Acceleration	**Magnitude [rad/s²]**
α_x	-3.4722e-5
α_y	3.4722e-5
α_z	−3.4722e-5

Table 3 Angular Acceleration

By using equation (3), we multiply the moment of inertia for our CubeSat with the desired angular acceleration and we find the minimum torque required to rotate the satellite to be summarized in Table 4 Minimum Torque

Torque	**Magnitude [Nm]**
τ_x	-1.157e-6
τ_y	1.157e-6
τ_z	-2.3148e-7

Table 4 Minimum Torque

Lastly, by rearranging the equation in equation (2), we can solve for the magnetic moment. The formulae are derived in Appendix A summarized in Table 5 Magnetic Moment Calculations.

Magnetic Moment Calculation	Formula
μ_x	$[\tau_z + \frac{B_{Ex}}{B_{Ez}}(\tau_x + \mu_z B_{Ey})]\frac{1}{B_{Ey}}$
μ_y	$(\tau_x + \mu_z B_{Ey})\frac{1}{B_{Ez}}$
μ_z	$\frac{\left[\tau_y + \frac{\tau_z}{B_{Ey}} + \frac{B_{Ex}}{B_{Ez}B_{Ey}}\tau_x\right]}{B_{Ex} - \frac{B_{Ex}}{B_{Ez}}}$

Table 5 Magnetic Moment Calculations

Notice μ_x and μ_y are in terms of μ_z, so to solve these equations we simply plug the previously calculated values into μ_z first and then back substitute. The calculations were executed by Matlab and the code is in Appendix B.

Based on a slew rate of less than 4 minutes and the minimum torque listed in Table 4 Minimum Torque the minimum magnitude of the magnetic moment for our design is **0.1301 Nm/T.**

Core Calculation

To maximize the torque generated by the magnetorquer, we must maximize the magnetic field and in turn the magnetic moment. The equation for a solenoid is below, where B is the magnetic field, k is the relative permeability, $\mu_0$is the vacuum permeability, N is the number of turns, L is the length of the core and I is the current. [13]

$$B = k\mu_0 \frac{N}{L} I \quad (12)$$

The first step is to determine the parameters of the core in terms of the mass constraint previously defined and the relations are derived in [3] and summarized in (13), where r_c, r_w, ρ_c and ρ_w are the cross-sectional areas and density of the core and wire respectively, l_c is the length of the core, M is the constrained mass, V is the voltage, P is the power consumption, W_w is the wire resistivity and N_d is the demagnetizing factor.

$$r_c{}^2 l_c = \frac{M}{\pi\rho_c} - \frac{\rho_w}{\rho_c}\frac{r_w V^2}{PW_w}$$

$$N_d = \frac{4[ln\left(\frac{l_c}{r_c}\right)-1]}{\left(\frac{l_c}{r_c}\right)^2 - 4\,ln(\frac{l_c}{r_c})} \quad (13)$$

$$\mu = \frac{r_c V}{2W_w}[1 + \frac{\mu_r - 1}{1 + (\mu_r - 1)N_d}]$$

By setting the limits for the length and radius of the core, we can create a range of values and then run them through an optimization formula to maximize the magnetic moment. By using the steepest ascent method, [14] we can visually plot where the optimal length and radius of the core will return the maximum magnetic moment. For the initial calculation, the assumption was the wire would be copper and the core would be iron.

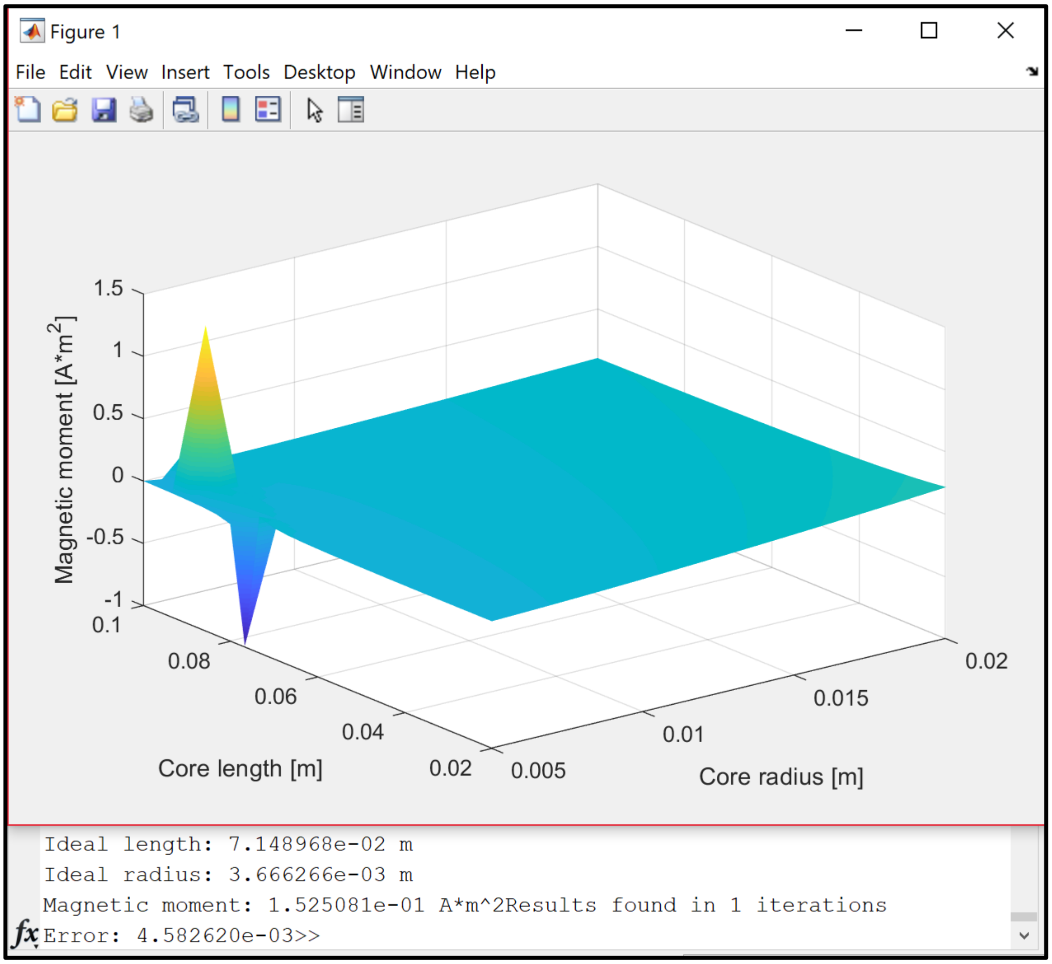

Figure 2 Ideal core length and radius

From the calculations made in Appendix C, the ideal length of the core is 7.15cm and radius is 3.67mm.

Next, we constrain our calculations to a length and radius that is available for purchase, but as close to the ideal values as possible. After running through a variety of options, the final selection was a 78ROD made by Fair-Rite Products Corporation [15] and a copper wire of gauge 34 from CnC Tech [16] which is graphed below.

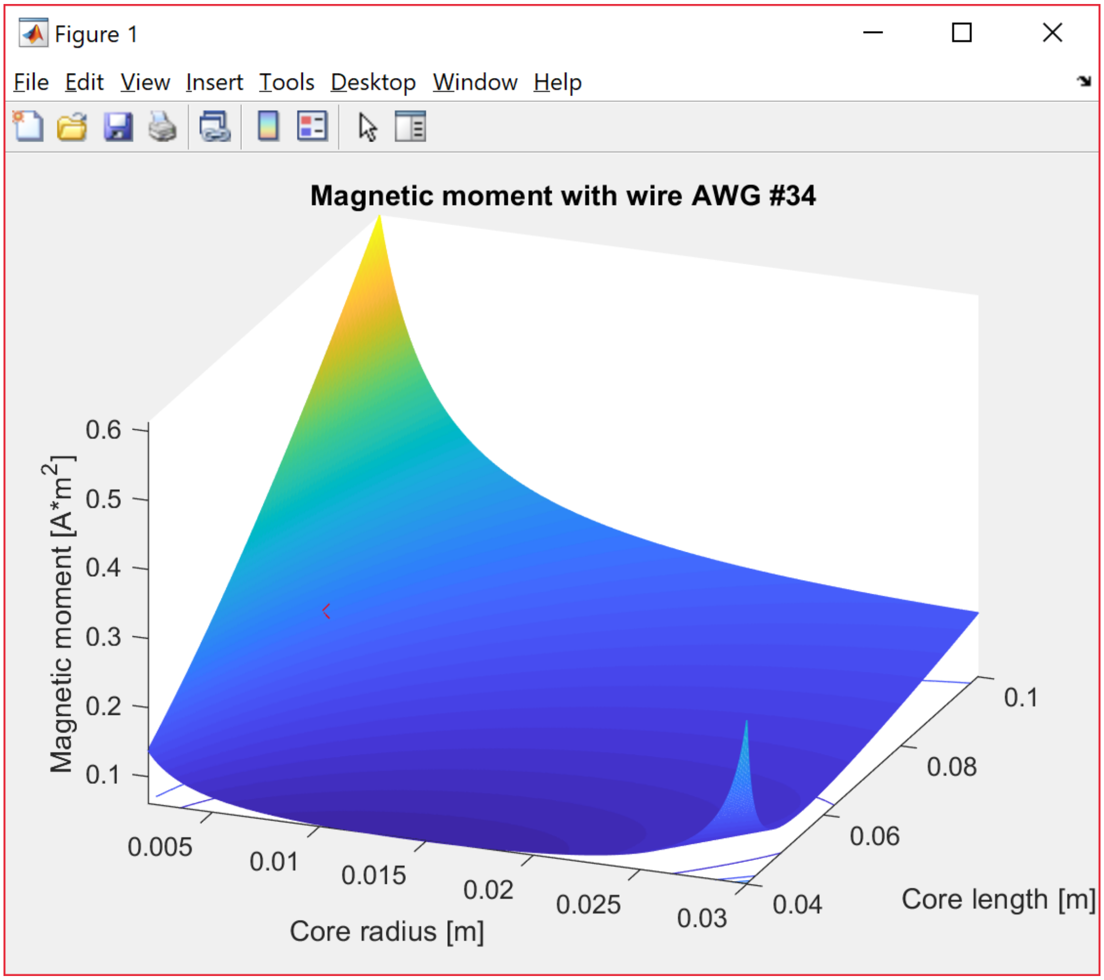

Figure 3 Actual magnetic moment from core radius and length

For our magnetorquer design, using the materials listed above, we can generate a magnetic moment of **0.1973 Nm/T**, which will require about 62.44m of wire and 2092 turns. The total mass will be 38.55gr for the core and wire combined and will run on 3.3V and consume about 0.2W. The code is summarized in Appendix D and the final design is summarized in Table 6 Final Design Parameters

Parameter	**Value**
Wire	AWG 34 – Copper
Wire Length	62.44m
Core Length	0.07m
Core Radius	0.00475m
Turns	2092
Total mass	38.55gr
Voltage	3.3V
Power	0.2W
Magnetic Moment	0.1973Nm/T

Table 6 Final Design Parameters

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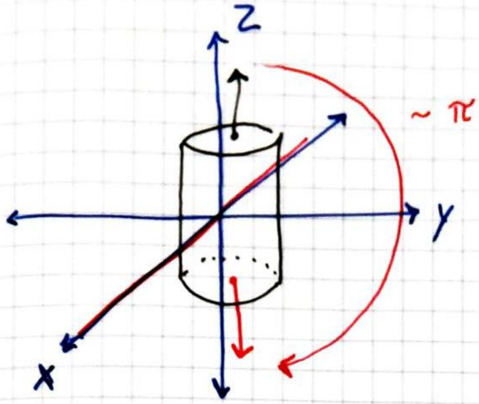
Appendix A



~ π only in one plane: only x component of angular acceleration is π! Otherwise, you will be rotating about all 3 axes.

$$\mu_y B_z - \mu_z B_y = \tau_x$$
$$\mu_z B_x - \mu_x B_z = \tau_y$$
$$\mu_x B_y - \mu_y B_x = \tau_z$$

① $\mu_y = (\tau_x + \mu_z B_y)\frac{1}{B_z}$

$$\mu_x B_y - (\tau_x + \mu_z B_y)\frac{B_x}{B_z} = \tau_z$$

② $\mu_x = \left[\tau_z + \frac{B_x}{B_z}(\tau_x + \mu_z B_y)\right]\frac{1}{B_y}$

$$\mu_z B_x - \left[\tau_z + \frac{B_x}{B_z}(\tau_x + \mu_z B_y)\right]\frac{1}{B_y} = \tau_y$$
$$\mu_z B_x - \tau_z / B_y - \frac{B_x}{B_z B_y}(\tau_x + \mu_z B_y) = \tau_y$$
$$\mu_z\left(B_x - \frac{B_x}{B_z}\right) = \tau_y + \frac{\tau_z}{B_y} + \frac{B_x}{B_z B_y}\tau_x$$

③ $\mu_z = \left[\tau_y + \frac{\tau_z}{B_y} + \frac{B_x}{B_z B_y}\tau_x\right] / \left(B_x - \frac{B_x}{B_z}\right)$

Appendix B

```
clear; clc;
format long;

%%%%%% PARAMETERS
 % Max time desired for rotation
 t = 4*60; %[s]

% Satellite dimensions
l_x = 0.1; %[m]
l_y = 0.1; %[m]
l_z = 0.3; %[m]

% Mass
m = 4; %[kg]

% Maximum rotation x,y,z
theta = pi(); %[rad]

% Earths magnetic field at the equator (at 25*S 25*W)
Be_x = 12654e-9; % [T]
Be_y = -6070e-9; % [T]
Be_z = -20527e-9; %[T]

%%%%% CALCULATION
% Moment of inertia
moment_inertia = [(l_y^2 + l_z^2), 0, 0
            0, (l_x^2 + l_z^2), 0
            0, 0, (l_x^2 + l_y^2)]*(m/12); %[kg*m^2]

% Angular acceleration
ang_accel = [cos(theta), 0, sin(theta)
             0, 1, 0
             -sin(theta), 0, cos(theta)]*(2/(t^2)); %[rad*s^-2]

torque =  torque_calc(moment_inertia, ang_accel); % [Nm]
```

```
% Pull out variable names for convenience
T_x = torque(1);
T_y = torque(2);
T_z = torque(3);

% magnetic moment components
mu_z = (T_y + T_z/Be_y + Be_x*T_x/(Be_z * Be_y))/(Be_x - Be_x/Be_z);
mu_y = (T_x + mu_z*Be_y)/Be_z;
mu_x = (T_z + (T_x + mu_z*Be_y)*Be_x/Be_z)/Be_y;

magnitude = sqrt(mu_x^2 + mu_y^2 + mu_z^2);

% Summary:
fprintf('mu_x = %f\nmu_y = %f\nmu_z = %f\n\n', mu_x, mu_y, mu_z);
fprintf('For a constant magnetic moment with magnitude %f, the satellite can rotate 180 degrees about the y axis in about %i minutes.\n', magnitude, t/60);
fprintf('Note: remember that this is the MINIMUM magnetic moment required to perform this maneuver.\n');

%%%%% FUNCTIONS
function [tau_vec] = torque_calc(I, ang_accel)

% Constant torque required to rotate the satellite
torque = I*ang_accel;

% Break it up into components
tau_vec(1,1) = torque(1,1);
tau_vec(2,1) = torque(2,2);
tau_vec(3,1) = torque(3,3);

end
```

Appendix C

```
clear; clc; close all;

Copper = 1360; %[Wire resistance]

W_res = Copper/1000;  %[Wire resistance, ohm/m]
mu_rod = 2000; %[Relative permeability of the rod]

%Range check
L = linspace(0.02, 0.1, 25); %[Length of rod, m] (2cm-10cm)
r = linspace(0.005, 0.02, 25); %[radius of rod, m] (0.5cm-2cm)

V_bus = 3.3; %[v]
P_max = 0.2;  %[W]

R_min = V_bus^2/P_max; %[ohm]
l_wire = R_min./W_res;

N = l_wire./(2*pi()*r); %[turns]

Z = zeros(length(r),length(L));

syms x y;
f =  x*V_bus/(2*W_res)*(1+(mu_rod-1)/(1+(mu_rod-1)*((4*log(y/x)-1)/((y/x)^2-4*log(y/x)))));
dx = diff(f,x);
dy = diff(f,y);
ddx = diff(dx,x);
ddy = diff(dy,y);
dxdy = diff(dx, y);
dydx = diff(dy, x);

%Plot the original function
[x1 y1] = meshgrid(r,L);
tol = 10^-4;
for i = 1:length(r)
  for j = 1:length(L)
    Z(i,j)=evaluate(f, r(i), L(j));
    fprintf('Loop i: %i, Loop j: %i\n', i, j);
  end
end
```

```
surf(x1, y1, Z);
shading interp;
xlabel('Core radius [m]')
ylabel('Core length [m]')
zlabel('Magnetic moment [A*m^2]')

X = [0.008;0.07]; %Start point

[X,mag,k,Err] = sam(X,tol, f, dx, dy, ddx, ddy, dxdy, dydx);

fprintf('Ideal length: %d m\nIdeal radius: %d m\nMagnetic moment: %d A*m^2', X(2, end), X(1, end),
evaluate(f, X(1, end), X(2, end)));
fprintf('Results found in %i iterations\nError: %d', k, Err(end));

%Steepest ascent method
function [X,fun,k,Err] = sam(X, tol, f, dx, dy, ddx, ddy, dxdy, dydx)
k = 0;
d = 1;
x = X;
fun(1) = evaluate(f,x(1),x(2));
Err = NaN;
while d > tol
   fprintf('Loop k: %i\n', k);
   xgrad = grad(x, dx, dy); %Evaluate gradient
   Hx = hess(x, ddx, dxdy, dydx, ddy); %Evaluate Hessian
   h = abs((xgrad'*xgrad)/(xgrad'*Hx*xgrad)); %Step size
   xnew = x+xgrad*h; %Evaluate steepest ascent method
   X = [X xnew];
   k = k+1;
   d = norm(xnew-x);
   Err = [Err d];
   x = xnew;
   fun(k+1) = evaluate(f,x(1),x(2));

   if (x(1) < 0.005 || x(2) < 0.02 || x(2) > 0.1)
      break
   end
end
end

function result = evaluate(f,x_val,y_val)
```

```
syms x y;
result = eval(subs(eval(subs(f,x,x_val)), y,y_val));
end

function xgrad = grad(X, dx, dy)

x = X(1);
y = X(2);

result_dx = evaluate(dx, x, y);
result_dy = evaluate(dy, x, y);

xgrad = [result_dx, result_dy]';
end

function Hx = hess(X, ddx, dxdy, dydx, ddy)

x = X(1);
y = X(2);


result_ddx = evaluate(ddx, x, y);
result_ddy = evaluate(ddy, x, y);
result_dxdy = evaluate(dxdy, x, y);
result_dydx = evaluate(dydx, x, y);

Hx = [result_ddx, result_dxdy; result_dydx, result_ddy];
end
```

Appendix D

```
clear; clc; close all;
% Wire selection from data sheets
%Digikey part # 1175-1716-ND $134.55
 AWG = 34;
 Copper = 872;
 wire_r = 0.00009;

% Rod selection from data sheets
% https://www.i-components.fi/components/Fair-Rite-Products-Corp/4078377511.html
% $1.008/rod
 mu_rod = 2300;
 p_core = 4900;
 L = 0.07;
 r = 0.00475;

%%%DEFINED CONSTRAINTS%%%
L_constrained = 0.07; %[m]
mass = 0.03; %[kg]
V_bus = 3.3; %[V]
P_max = 0.2;  %[W]

wire = Copper;
p_wire = 8960;
W_res = wire/1000;  %[Wire resistance, ohm/m]

R_min = V_bus^2/P_max; %[ohm]
l_wire = R_min./W_res; %[m]

m_wire = pi()*wire_r^2*l_wire*p_wire;
m_rod = pi()*r^2*L*p_core;
m = m_wire + m_rod;
turns = l_wire/(2*pi()*r);

mag_moment = r*V_bus/(2*W_res)*(1+(mu_rod-1)/(1+(mu_rod-1)*demag(L,r)));

fprintf('For a wire gauge of %.f, we will need about %.2fm of wire. This means %.f turns. \n', AWG, l_wire, turns);
fprintf('The total mass will be %.2fgr\n', m*1000);
fprintf('The maximum magnetic moment with %.2fV and %.3fW is %.4f', V_bus, P_max, mag_moment);
function [demagnetization] = demag(L,r)
demagnetization = (4*log(L/r)-1)/((L/r)^2-4*log(L/r));
end
```